Thermofluids 3

Exercise Sheet Gas Turbines - SOLUTIONS

1) a) Simple Cycle

In compressor

$$T_2' = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$T_1 = 15°C = 288K$

$\gamma = 1.4$

$$= 288 \, (10)^{\frac{0.4}{1.4}}$$

$$= 566.0$$

$$\Delta T' = 268.0$$

$$\eta_{c\,isen} = 0.85 = \frac{\Delta T'}{\Delta T}$$

$$\Delta T = 315.3 \text{ K.}$$

$$\therefore T_2 = 603.3K \quad (330.3\ °C)$$

$$w_{comp} = c_p \Delta T = 1.005 \times 315.3$$

$$= \underline{316.9 \text{ kJ/kg.}}$$

$c_p = 1.005$ kJ/kgK

In turbine

$$T_4' = T_3 \left(\frac{p_4}{p_3}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}$$

$T_3 = 1100$ K

$$= 1100 \left(\frac{1}{10}\right)^{\frac{0.4}{1.4}}$$

$$T_4' = 569.7 \text{ K} \qquad \Delta T' = 530.3$$

$$\eta_{t\,isen} = 0.90 = \frac{\Delta T}{\Delta T'}$$

$$\Delta T = 530.3 \times 0.90$$

$$= 477.3 \text{ K}$$

$$T_4 = 622.7 \text{ K} \quad (349.7\ °C)$$

$$w_{turbine} = c_p \Delta T$$

$$= \underline{479.7 \text{ kJ/kg}}$$

Net work output $= 479.7 - 316.9$

$w_{net} = 162.8$ kJ/kg.

Energy input $= c_p(T_3 - T_2) = 1.005\,(1100 - 603)$

$= 499.2$ kJ/kg

Cycle efficiency $= \dfrac{w_{net}}{\text{heat input}} = 0.326$

$\eta_{GT} = 32.6\%$

Work ratio $= \dfrac{w_{net}}{w_{turb}}$

$= 0.339$

b) Heat Exchange Cycle

$T_2' = 288\,(6)^{\frac{0.4}{1.4}} = 480.5$ K

$\Delta T' = 192.5$

$\Delta T = \dfrac{\Delta T'}{0.85} = 226.5$

$T_2 = 514.5\text{ K}\ (241.5\,^\circ\text{C})$

$w_{comp} = 1.005 \times 226.5$

$w_{comp} = 227.6$ kJ/kg

$T_4' = 1100\left(\frac{1}{6}\right)^{\frac{0.4}{1.4}} = 659.3$ K

$\Delta T' = 440.7$ $\qquad \Delta T = 440.7 \times \eta_{isen}$

$\Delta T = 396.6$

$T_4 = 703.4$ K

$w_{turb} = c_p \Delta T = 398.6$ kJ/kg

Heat exchange effectiveness $= 0.9 = \dfrac{T_{2a} - T_2}{T_4 - T_2}$

$T_{2a} = 684.5 \text{ K}$

Energy input $= C_p(1100 - 684.5) = 415.5 \text{ kJ/kg}$

Net work output $= 398.6 - 227.6 = 171 \text{ kJ/kg}$

Work ratio $r_w = \dfrac{171}{398.6} = 0.429$

Efficiency $= \dfrac{171}{415.5} = 41.2\%$

Exhaust temp $= T_{4a}$

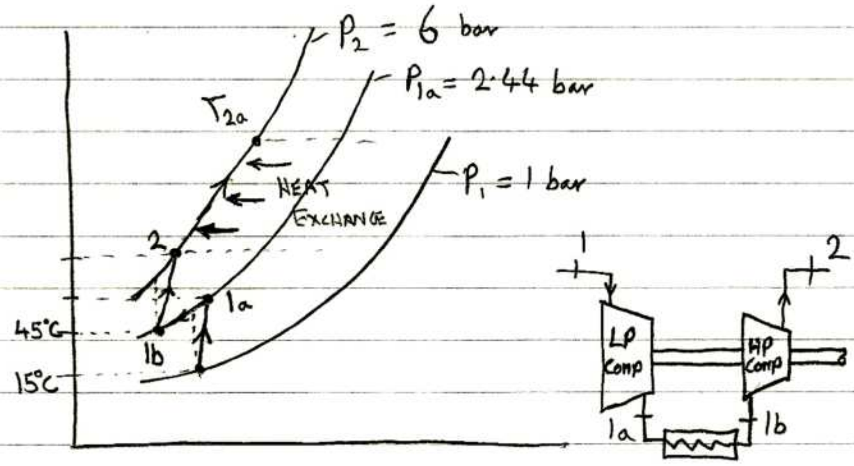
Heat gain in compressor air = heat lost by exhaust in recuperator

$\therefore (T_{2a} - T_2) = (T_4 - T_{4a})$

$(684.5 - 514.5) = (703.4 - T_{4a})$

$T_{4a} = 533.4 \text{ K} \quad (260.4°C)$

1(c) Intercooled Heat Exchange Cycle



In 1st stage of compressor

$$T_{1a}' = T_1 \left(\frac{p_{1a}}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \qquad \frac{\gamma-1}{\gamma} = 0.286$$

$$= 288\,(2.44)^{0.286}$$

$$T_{1a}' = 371.7 \text{ K}$$

$$\Delta T_{1-1a}' = 83.7 \text{ K}$$

$$\Delta T_{1-1a} = \frac{\Delta T_{1-1a}'}{0.85}$$

$$\Delta T_{1-1a} = 98.5 \text{ K}$$

$$T_{1a} = 386.5 \text{ K} \quad (113.5^\circ C)$$

In second stage of compressor $T_{1b} = 45^\circ C \quad (318 \text{ K})$

$$T_2' = T_{1b} \left(\frac{p_2}{p_{1a}}\right)^{\frac{\gamma-1}{\gamma}} = 318\,(2.44)^{0.286}$$

$$T_2' = 410.4 \text{ K}$$

$$\Delta T'_{1b-2} = 92.4$$

$$\Delta T_{1b-2} = \frac{92.4}{0.85} = 108.7$$

$$T_2 = T_{1b} + \Delta T_{1b-2} = 426.7\,K \quad (153.7\,^\circ C)$$

Work input to compressor (both stages)

$$W_c = C_p\left((T_{1a} - T_1) + (T_2 - T_{1b})\right)$$

$$= 1.005\left((113.5 - 15) + (153.7 - 45)\right)$$

$$W_c = 208.2 \text{ kJ/kg}$$

Outlet temp from turbine T_4 will not change, so it will be 703.4K as in part (b).

$$\text{Effectiveness of heat exchanger} = \frac{T_{2a} - T_2}{T_4 - T_2} = 0.9$$

$$T_{2a} = 0.9(703.4 - 426.7) + 426.7$$

$$T_{2a} = 675.7\,K \quad (402.7\,^\circ C)$$

Rise in temperature of compressed air across heat exchanger will be same as drop in temperature of exhaust, as mass flows are same.

$$\text{Exhaust temp} = T_4 - (T_{2a} - T_2) = 703.4 - (675.7 - 426.7)$$

$$\text{Exh Temp} = 454.4\,K \quad (181.4\,^\circ C)$$

Work output from turbine is same as part(b) $W_{turb} = 398.6$ kJ/kg

Net (specific) work output $= (398.6 - 208.2) = 190.4$ kJ/kg

(GT-6)

Work ratio $= \frac{190.4}{398.6} = 0.48$

Heat input in combustion chamber $= c_p (T_3 - T_{2a})$

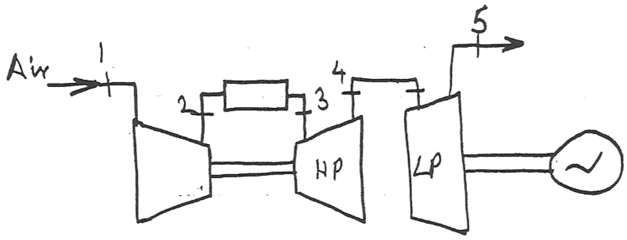
$= 1.005 (1100 - 675.7)$

$Q_{in} = 426.4 \text{ kJ/kg}$

Cycle efficiency $= \frac{w_{net}}{Q_{in}} = \frac{190.4}{426.4}$

$\eta_{GT} = 44.7\%$

2)



$\dot{m} = 80$ kg/s

$\gamma = 1.4$ $c_p = 1.004$ (compressor)

$T_1 = 288$ K

$$T_2' = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 556.0$$

$$\Delta T' = 268.0$$

$$\Delta T = 315.3$$

$$\omega_{comp} = 1.004 \times 315.3 = 316.6 \text{ kJ/kg}$$

Assume no losses in shaft, $\omega_{comp} = \omega_{turbine\ HP}$.

$$\omega_{turbine\ HP} = 316.6 = 1.15(\Delta T)$$

$\gamma = 1.333$

$c_p = 1.15$ kJ/kgK

$$\Delta T_{HP} = 275.3 \qquad T_4 = 874.7 \text{ K}$$

$$\Delta T'_{HP} = \frac{\Delta T}{\eta_{isen\ turb\ HP}} = 312.8 \text{ K}$$

$$T_4' = 837.2 \text{ K}$$

$$\frac{T_3}{T_4'} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}}$$

$$p_4 = p_3 \left(\frac{T_4'}{T_3}\right)^{\frac{\gamma}{\gamma-1}} = 2.808 \text{ bar}$$

$$T_5' = T_4 \left(\frac{p_5}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = 874.7 \times \left(\frac{1}{2.808}\right)^{\frac{0.333}{1.333}}$$

$$= 675.7 \text{ K}$$

$$\Delta T' = 199.0$$

$$\Delta T = 179.1$$

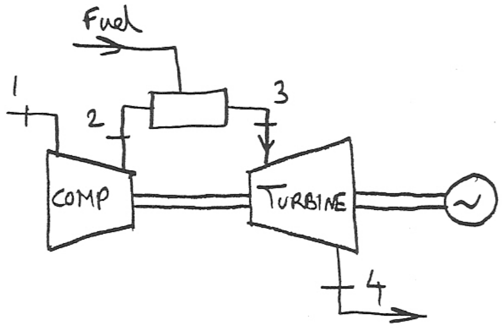
$$\omega_{turb\ LP} = 179.1 \times 1.15$$

$$= 205.97$$

$$W_{elec} = 205.97 \times 80 \times 0.91$$

$$= \underline{\underline{15.0 \text{ MW}}}$$

3)



Pressures

Pressure at compressor inlet $p_1 = 1.013 - 0.01$ bar $= 1.003$ bar

Pressure ratio is 15 $\therefore$ $p_2 = 15.045$ bar.

There is a 0.5 bar loss in the combustion chamber, $\therefore$ $p_3 = 14.545$ bar

The outlet pressure p_4 is 0.02 bar above atmosphere $\therefore$ $p_4 = 1.033$ bar.

Compressor — $$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{(\gamma-1)}{\gamma\eta_{\infty c}}}$$

In compressor assume $\gamma = 1.4$ & $c_p = 1.005$ kJ/kgK.

$$\frac{(\gamma-1)}{\gamma\eta_{\infty c}} = \frac{.4}{1.4\times0.85} = 0.3361$$

$$T_2 = 288\,(15)^{0.3361} = 715.6 \text{ K.} \quad (442.6\,^\circ C)$$

In combustion chamber — enthalpy balance

$$\dot{m}_f \times NCV = (\dot{m}_a + \dot{m}_f)\,c_{p\,exh}\,(1290-25) - \dot{m}_a c_{p\,air}(442.6-25)$$

$$\left(\frac{\dot{m}_f}{\dot{m}_a}\right) \times 42000 = \left(1 + \frac{\dot{m}_f}{\dot{m}_a}\right) 1.15\,(1265) - 1.005(417.6)$$

$$\left(\frac{\dot{m}_f}{\dot{m}_a}\right) = 0.02553 \qquad (0.0251 \text{ from Chart})$$

In turbine $$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\eta_{\infty t}(\gamma-1)}{\gamma}}$$

$$\eta_{\infty t}\left(\frac{\gamma-1}{\gamma}\right) = \left(\frac{0.85\times 0.333}{1.333}\right)$$

$$= 0.2125$$

$$\frac{T_3}{T_4} = 1.754 = \frac{1563}{T_4}$$

$$\therefore\ T_4 = 891.1\ \text{K}$$

Work input to compressor $= \dot{m}_a\, c_{p\,air}(T_2 - T_1)$

$$= \dot{m}_a\, 1.005(715.6 - 288)$$

$$= \dot{m}_a\, 429.74 \quad \text{kW}$$

In turbine, gross work output $= \dot{m}_a\left(1+\frac{\dot{m}_f}{\dot{m}_a}\right) c_{p\,exh}(T_3 - T_4)$

$$= \dot{m}_a(1.02553)\, 1.15(1563-891)$$

$$= \dot{m}_a\, 792.53 \quad \text{kW}$$

Electrical output $= \dot{m}_a(792.53 - 429.74)\times 0.96$

$$= \dot{m}_a\, 348.278 \quad \text{kW}$$

Actual output = 226.5 MW

$\therefore$ Air flow = 650.3 kg/s

Fuel flow rate = 16.60 kg/s   Fuel input energy = 697.3 MW

$\therefore$ Overall efficiency is 32.5 %